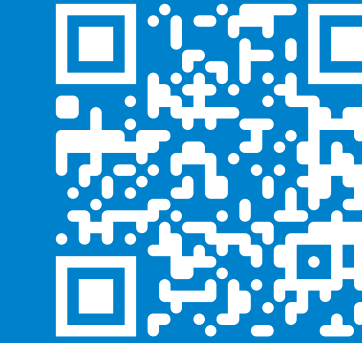


Understanding an SI second with more clocks

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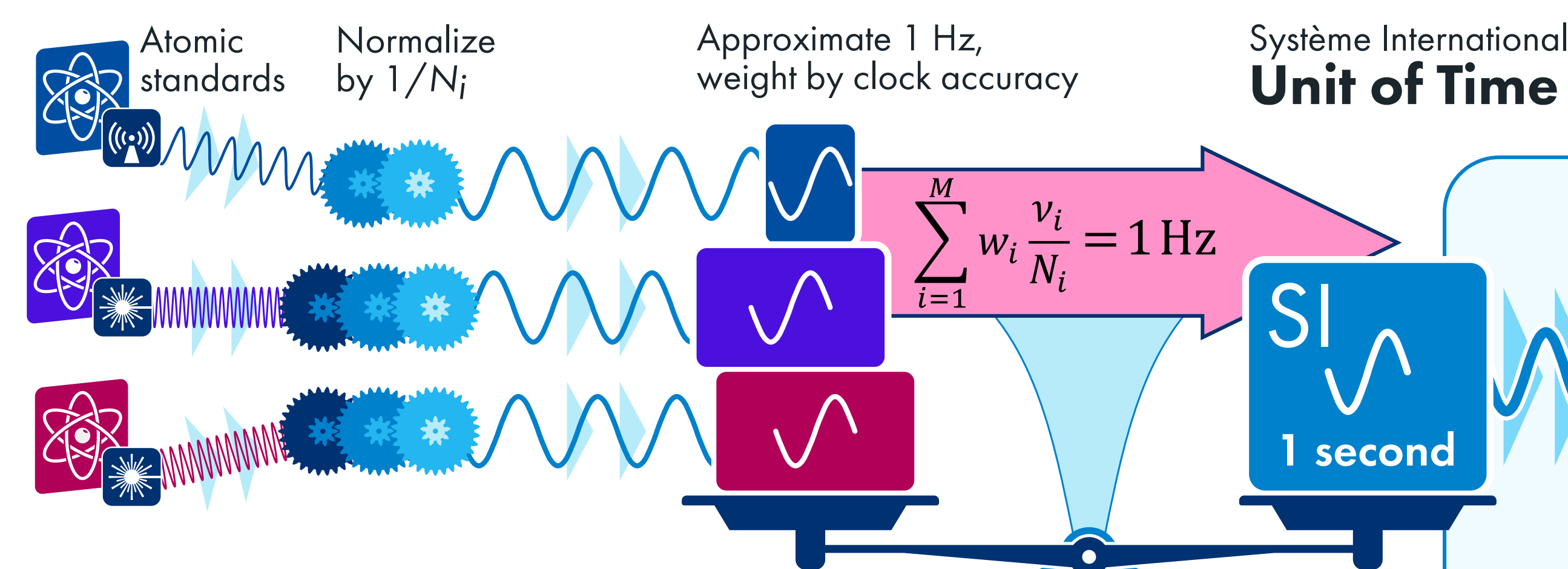
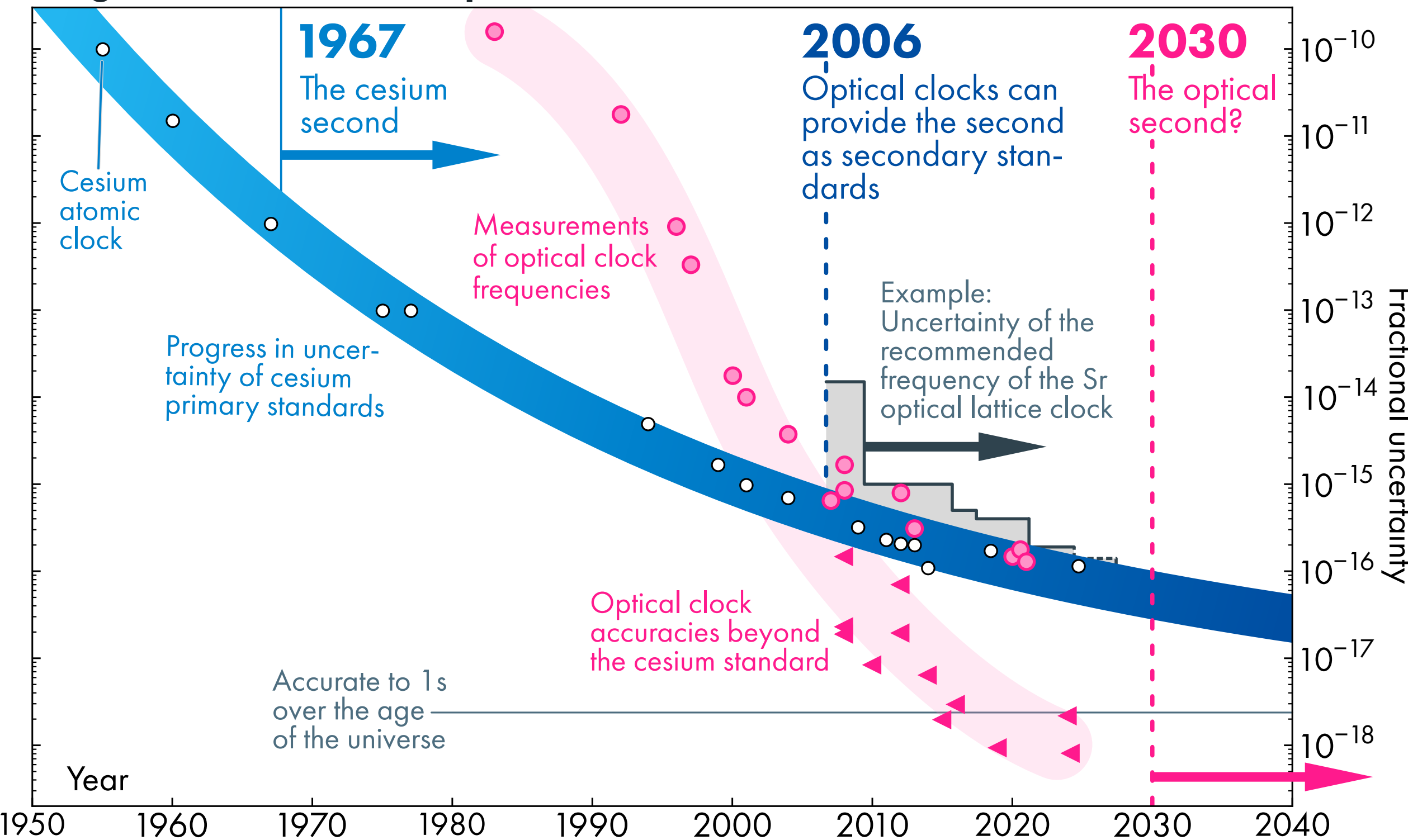
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Towards a redefinition of the second

Optical clocks now outperform the cesium standards that define the SI second since 1967. A redefinition is now planned for the CGPM meeting in 2030 [1]. But how do we choose a new definition that is as successful and long-lasting as the cesium second when clocks based on different transitions report competing measurement uncertainties close to 10^{-18} ?

Progress of cesium and optical clocks



Defining an ensemble SI second

The second, symbol s , is the SI unit of time. It is defined by fixing the weighted mean of M normalized frequencies ν_i that represent unperturbed atomic transitions as

$$\sum_{i=1}^M w_i \frac{\nu_i}{N_i} = 1 \text{ Hz} \quad \text{with weights} \quad \sum_{i=1}^M w_i = 1$$

The normalizations N_i are chosen to represent the ratios between the transition frequencies and the continuity with the previous definition. The assigned weights w_i reflect the accuracy of realizing each unperturbed frequency ν_i . Taken together, the tabulated constants ν_i and N_i provide an unambiguous and universal definition of the SI second.

Transition	Normalization constant N_i	Weight w_i
$^{27}\text{Al}^+$ ion, $1S_0 \leftrightarrow 3P_0$	1,121,015,393,207,859.157	1/3
neutral ^{87}Sr , $1S_0 \leftrightarrow 3P_0$	429,228,004,229,872.991	1/3
neutral ^{171}Yb , $1S_0 \leftrightarrow 3P_0$	518,295,836,590,863.600	1/3

Authors' note:
Numerical values are only for illustration and chosen from [3] as a convenient source.

Ensemble proposal

Option 1: Selecting the most promising type of optical clock as the **single primary standard** for the new SI second.

Other clocks act as secondary frequency standards, but are limited by the performance of the primary standard if it falls behind expectations. Development will focus on the primary standard, and others will receive less attention.

Option 2: Defining the new SI second with an ensemble of the best candidates [4] as **co-primary standards**. Averaging over more clocks and over different kinds of clocks is an intrinsic benefit to accuracy.

But can a single co-primary standard then accurately realize the SI second? Or would it always require access to the complete ensemble? Do we need perfect knowledge of the ratios of the ensemble transition frequencies to choose defining constants? Is an ensemble with arbitrary weights as universal, fundamental and real as a single chosen clock transition?

Discussing these questions is complicated by the mathematical structure of the current Option 2 proposal: Stated as a geometric mean, it merges all defining frequencies into a single value that is coupled to the assigned weights — weakening the physical significance.

We suggest an equivalent, more accessible form:

$$\prod_{i=1}^M \nu_i^{w_i} = N \text{ Hz} \quad \longrightarrow \quad \sum_{i=1}^M w_i \frac{\nu_i}{N_i} = 1 \text{ Hz}$$

Take-home points

What we describe here is a reformulation of the original proposal for an SI second with a set of optical clock transitions, presented by Jérôme Lodewyck in 2019 [4].

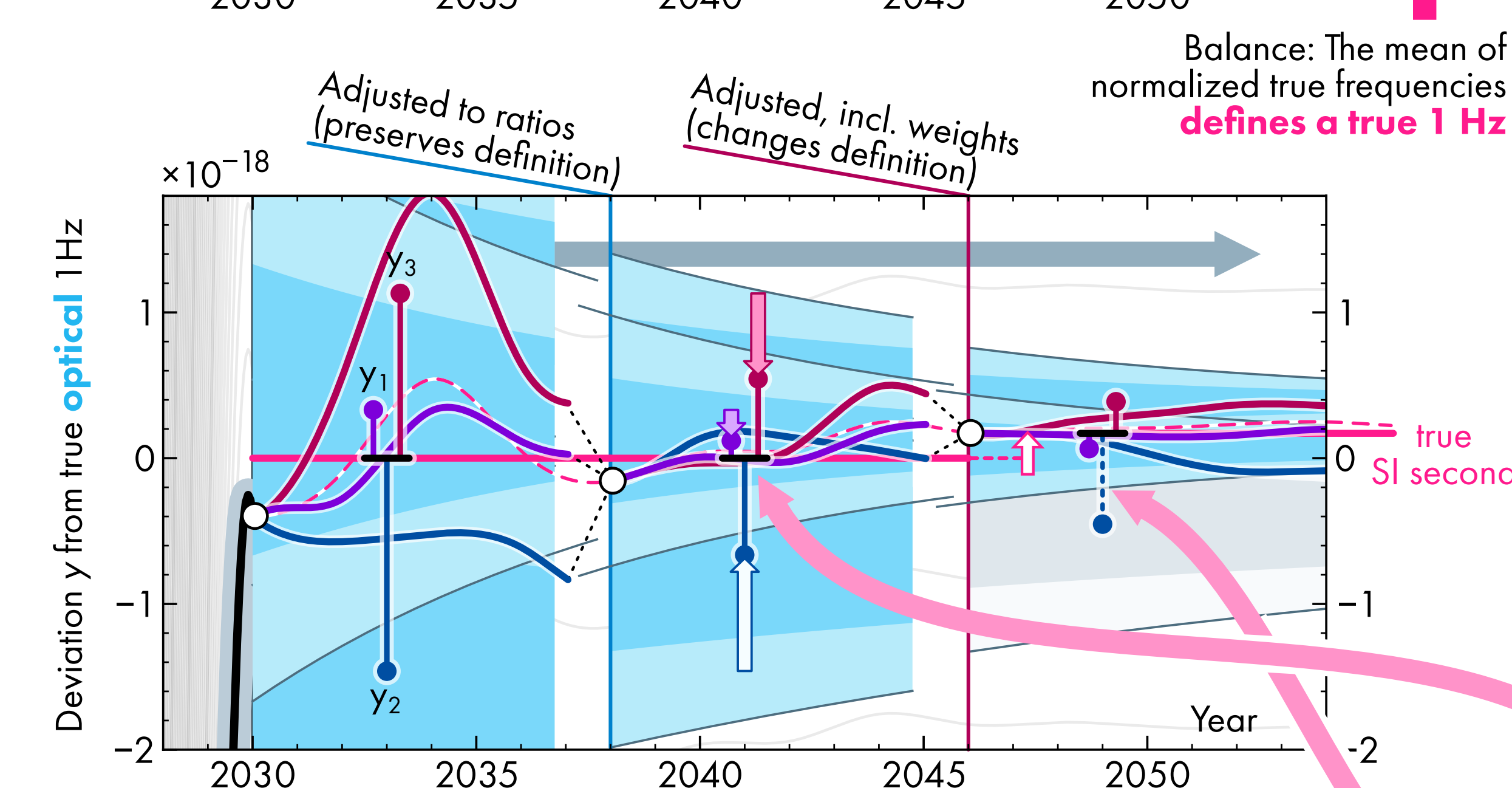
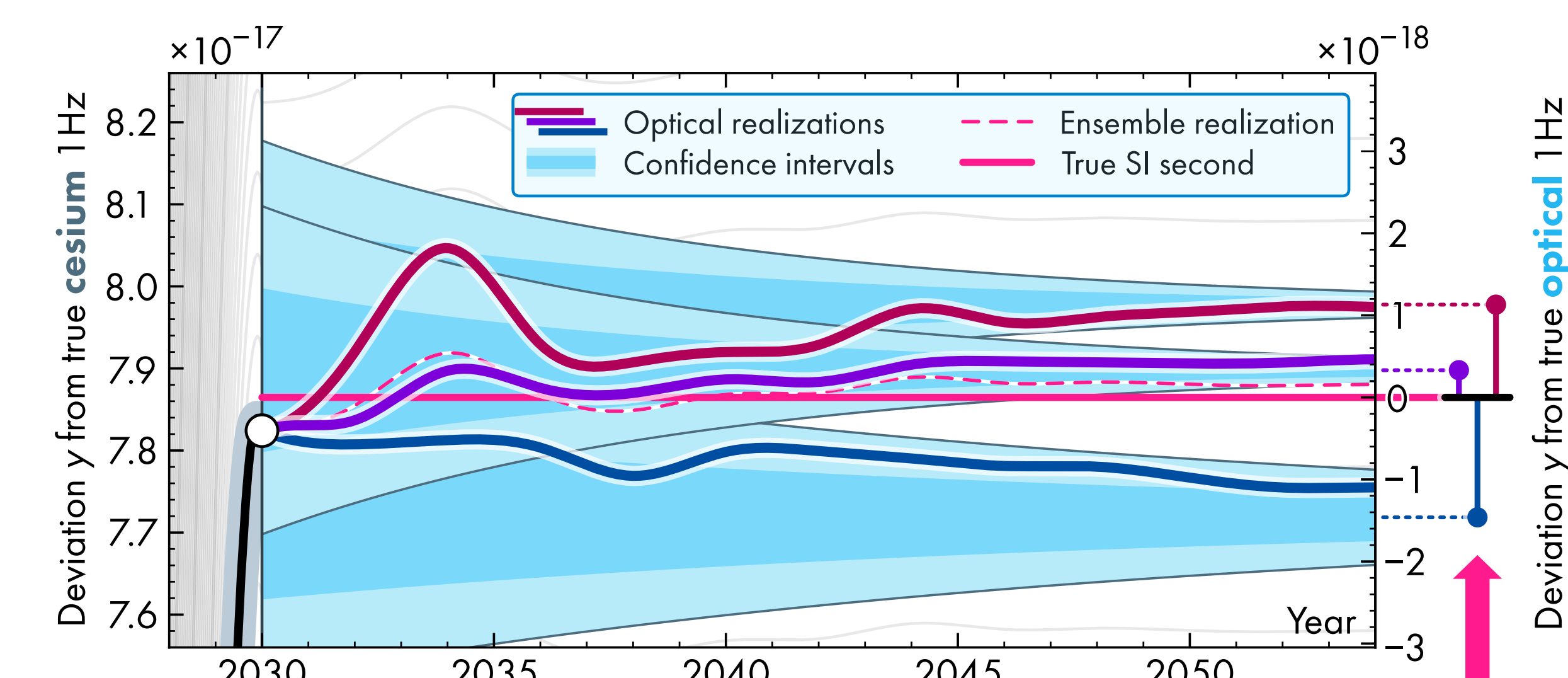
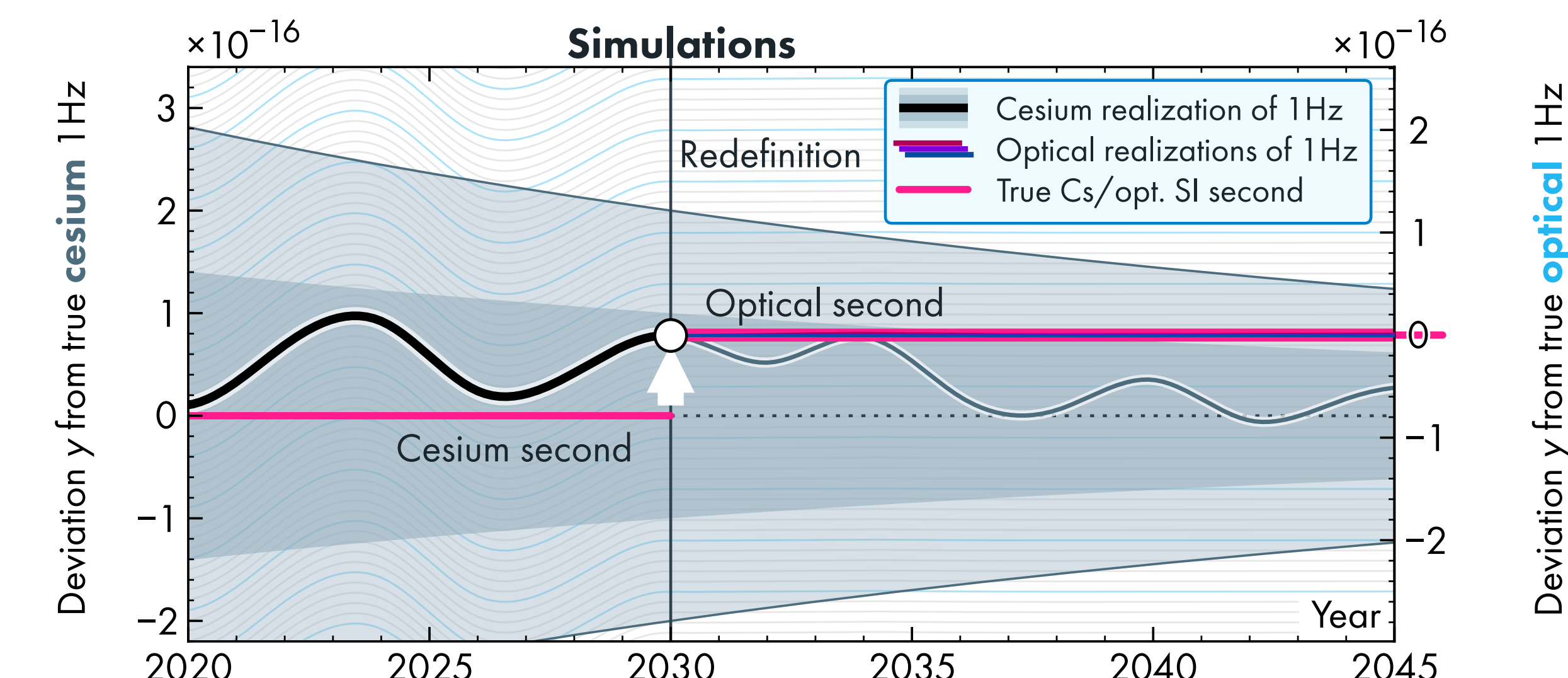
Our formulation emphasizes each contributing standard as the source of a normalized frequency that already represents the second to the limits of the standard's accuracy.

The SI second is then defined by the average of these frequencies, using weights chosen to make good use of each standard's performance over an extended period of continued development.

This new formulation may be viewed as a linearization of the original proposal. Working with frequency standards characterized to fractional uncertainties below 10^{-15} , it differs in second order terms of less than 10^{-30} . Therefore, all previous investigations are valid for either formulation.

An ensemble definition is a conservative approach that does not require a bet on which standard will achieve the greatest accuracy in the future. It avoids concentrating research on a specific standard and employs more of the existing optical clocks to realize the second.

We hope that it will be given consideration as a straightforward, intuitively accessible, competitive option for the upcoming redefinition of the SI second.



Co-primary realizations

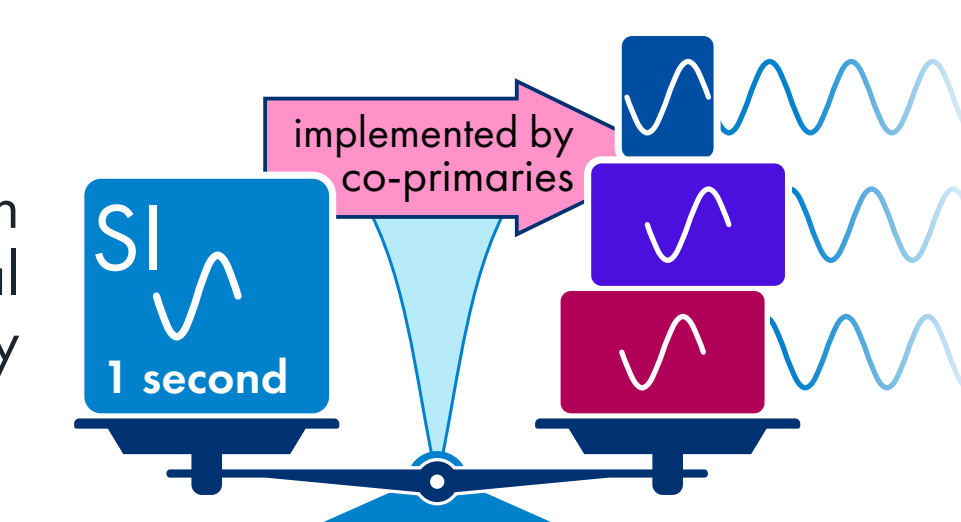
Any clock with a link to a full ensemble can take direct advantage of the weighted mean, potentially surpassing its own individual accuracy, or even that of the best single clock in the ensemble.

But the normalization constants, with published updates and uncertainty, turn the output of each clock implementing a part of the definition into a **co-primary realization** of the second: The ensemble SI second is a second with more clocks.

In a simple model the implementations of standard i compare to others with an uncertainty u_i when the frequency ratios are determined. A new clock of this type, with no link to the full ensemble, can then still realize the SI second with

$$u_{co,i}^2 = ([1 - w_i] u_i)^2 + \sum_{j \neq i} (w_j u_j)^2$$

as the uncertainty of a co-primary realization, in addition to the uncertainties of the clock itself. No individual standard limits the accuracy like it would for secondary realizations in a definition based on a single standard.



Evolution of the second

Introduction of an optical second

The normalized transition frequency ν_{CS}/N_{CS} defines a 1 Hz frequency that represents the cesium second. Our realizations of the second improve with better measurements, but the **true value** is not accessible.

When the optical SI second is introduced, it aims to leave the realization unchanged. Measurements performed in units of the realized second then continue to give identical results. But due to the uncertainties in the realizations, the inaccessible **true value** of the SI second changes at the instant of redefinition.

Zooming in on the optical standards

The normalization constants N_i are decided and fixed so that each **optical realization** matches the cesium realization:

$$N_i = r_{i,CS} N_{CS} \quad \text{aims to make} \quad \frac{\nu_i}{N_i} = \frac{\nu_{CS}}{N_{CS}}$$

The self-consistent, transitive matrix $r_{i,j} \approx \nu_i/\nu_j$ combines all available clock comparison data. A Working Group of the Consultative Committee for Time and Frequency already performs the required least-squares optimization [5,6]. Setting $N_i/N_j = r_{i,j}$ makes the optical realizations consistent beyond the limit of the cesium second.

Deciding the weights then **defines a true SI second** by the mean of the normalized transition frequencies:

$$\sum_{i=1}^M w_i \frac{\nu_i}{N_i} = 1 \text{ Hz}, \quad \text{equivalent to} \quad \sum_{i=1}^M w_i y_i = 0$$

for the fractional deviations $y_i = \nu_i/(N_i \text{ Hz}) - 1$.

Revising and reporting normalizations

The normalization constants that were decided and fixed reflect the frequency ratios only to the limit of information available at the time of the definition. As frequency standards improve, the **optical realizations** will eventually resolve the non-zero deviations y_i .

This is no limitation for the second: The ensemble mean continues to converge towards the **true SI second**. But realizations by single co-primary standards benefit from reflecting updated ratios $r_{i,j}$ as revised factors:

$$N'_i = \sum_{j=1}^M w_j r'_{i,j} N_j$$

This revision preserves the weighted mean: It does not affect the duration of the **true SI second**. Updated normalization factors may be published like recommended frequencies for secondary representations of the second are now.

Deciding the constants defines the ensemble SI second

Deciding and fixing N_i creates true normalized transition frequencies, but in contrast to the cesium second, they differ from 1 Hz due to our incomplete knowledge of the frequency ratios represented by $r_{i,j}$. The definition then locates the true SI second at the center of mass for non-zero deviations y_i that make $\sum_{i=1}^M w_i y_i = 0$. This means that any change to the weights redefines the SI second! The best weights follow to the familiar $w_i \propto u_i^{-2}$ [4], but u_i is now an estimate of the uncertainty for a varying mix of optical clocks that will probe the transition over the lifespan of the definition.

Where the normalization constants correspond to fundamental atomic qualities, the weights correspond to technical clock properties. But deciding and fixing either of these defining constants is an informed, but arbitrary act, just like the choice of the defining transitions.

[1] N. Dimarcq et al., Metrologia 61 012001, 2024
[2] H. Margolis, IEEE IFCS-UFC Joint Symposium, 2024

[3] K. Beloy et al., Nature 591 566–569, 2024
[4] J. Lodewyck, Metrologia 56 055009, 2019

[5] H. S. Margolis and P. Gill, Metrologia 52 628, 2015
[6] H. S. Margolis et al., Metrologia 61 035005, 2024