

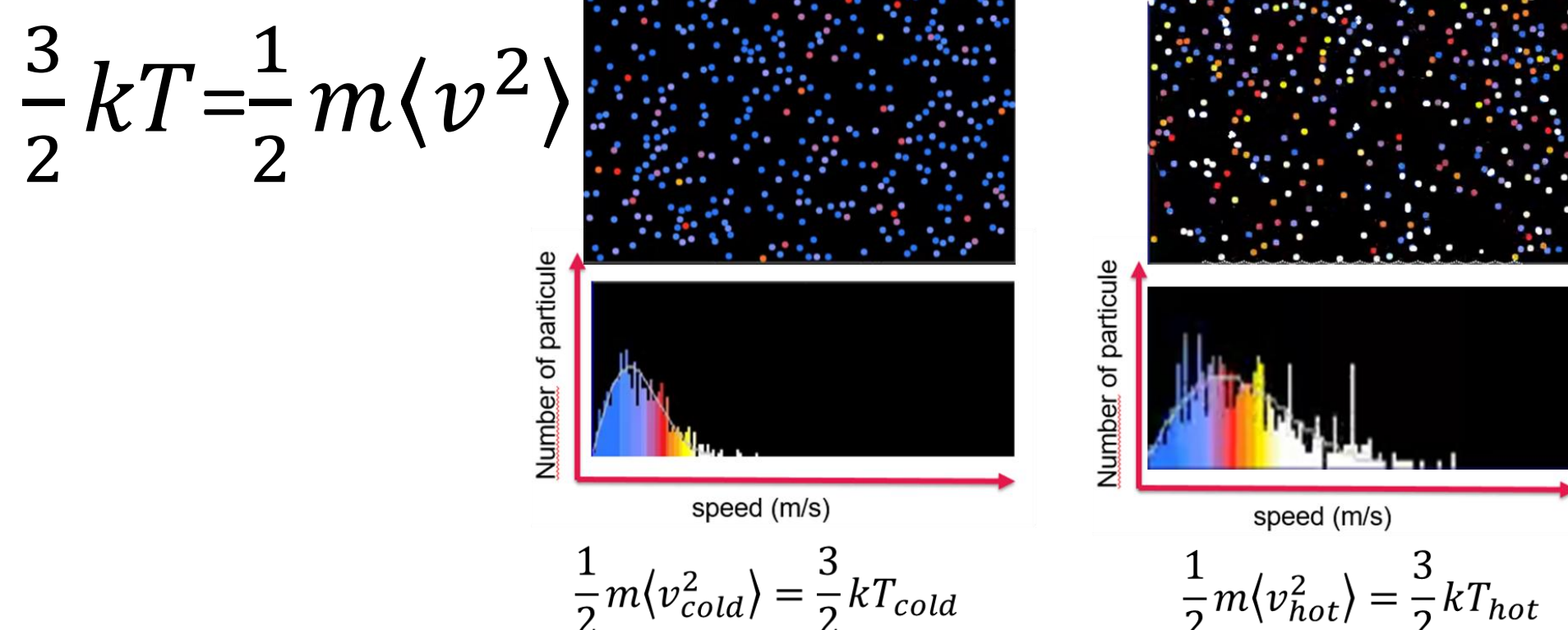
Introduction

Today, **over 50% of measurements worldwide involve thermometry**, and there is a growing demand for **traceability over extended periods** (centuries) or in environments where **recalibration** is impossible, such as **in space**. The establishment of a direct **link between the kelvin and a fundamental constant**, according to the 2019 definition, is already enabling new possibilities, where no particular temperature (e.g. the triple point of water) is more important than another and where thermometry techniques based on different physical principles can be used with equal merit. To meet these challenges, **LNE-Cnam determined k using acoustic thermometry with helium-4 gas in a 3-litre quasi-spherical resonator** (QSR), a tri-axial ellipsoid, **achieving the lowest uncertainty ever recorded for this measurement - an uncertainty that will remain unmatched, as the numerical value of k is now indefinitely fixed**.

The LNE-Cnam method involved measuring the speed of sound in helium gas by analyzing acoustic and microwave resonances. For an ideal gas at zero pressure, the speed depends only on k , the thermodynamic temperature T , the molar mass M and the Avogadro constant N_A . For maximal thermometric accuracy, the resonator was maintained at a stable temperature of 273.16 K corresponding to the triple point of water which defined the kelvin at the time. Acoustic resonances were measured at different pressures to allow extrapolation to zero pressure, while the three radii of the ellipsoid were determined *in situ* using microwave resonances. To guarantee the most accurate molar mass, the helium gas was continuously purified to remove contaminants, the tiny amount of the only unavoidable impurity, helium-3 gas being quantified using mass spectrometry on samples from the same source as the experiment.

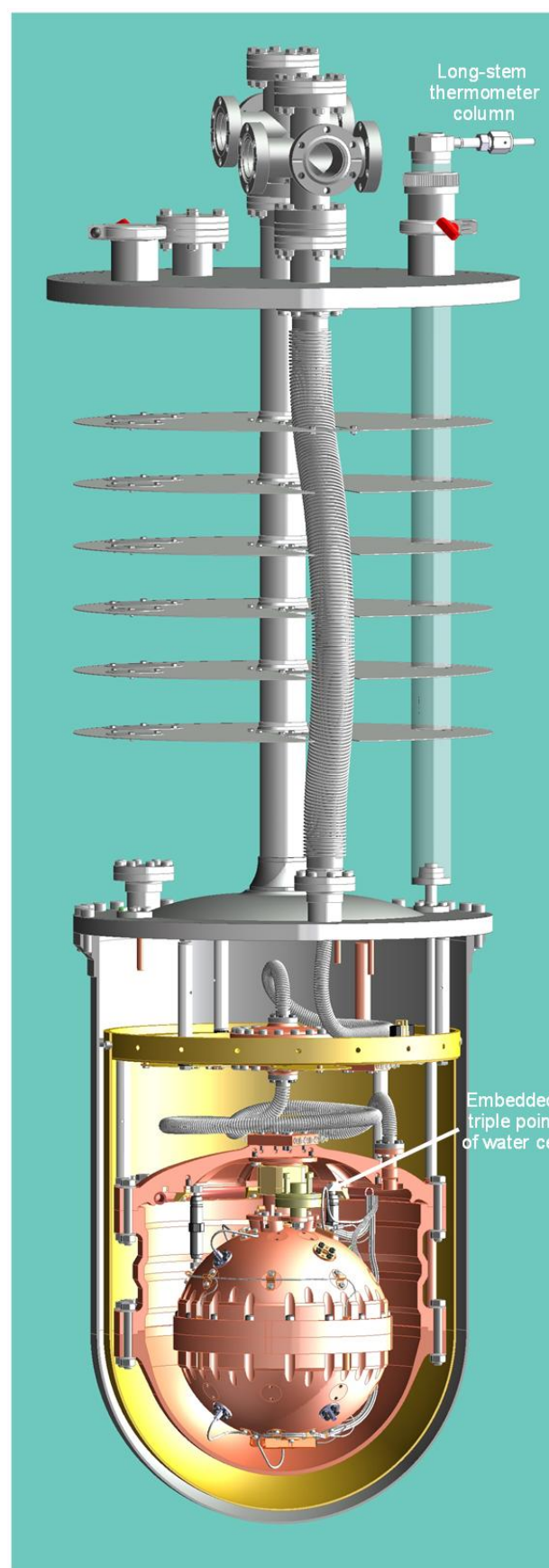
The kelvin and the Boltzmann constant

At thermal equilibrium, the thermal energy of gas molecules can be directly related to their kinetic energy by explicitly linking the **Boltzmann constant k** , the **temperature** of the gas T , and the **average squared speed** $\langle v^2 \rangle$ of the molecules.



Determination of k at LNE-Cnam with a 3.1 litre quasi sphere

1: The LNE-Cnam apparatus

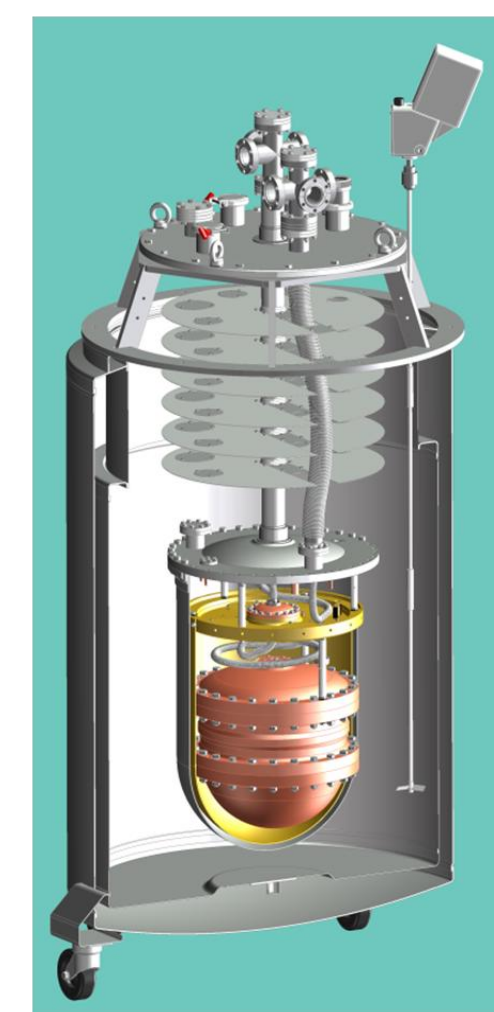
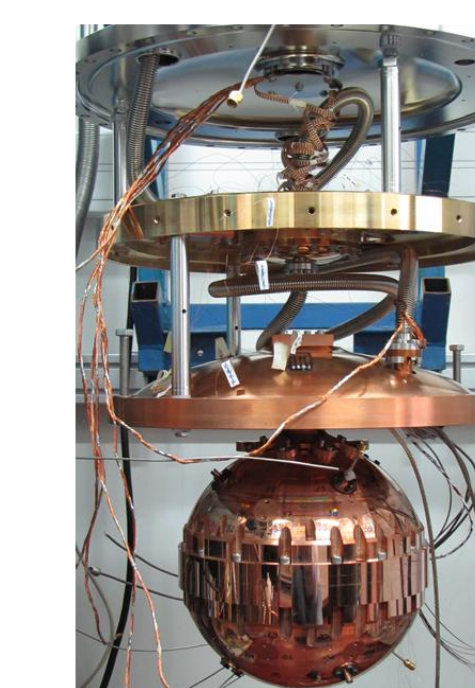


The thermostat

- Adiabatic thermostat from 77 K to 400 K
- 5 capsule-tyl and 2 standard platinum resistance thermometers
- Pressure vessel up to 7 bar
- No thermal gradient around the sphere
- No mechanical vibration

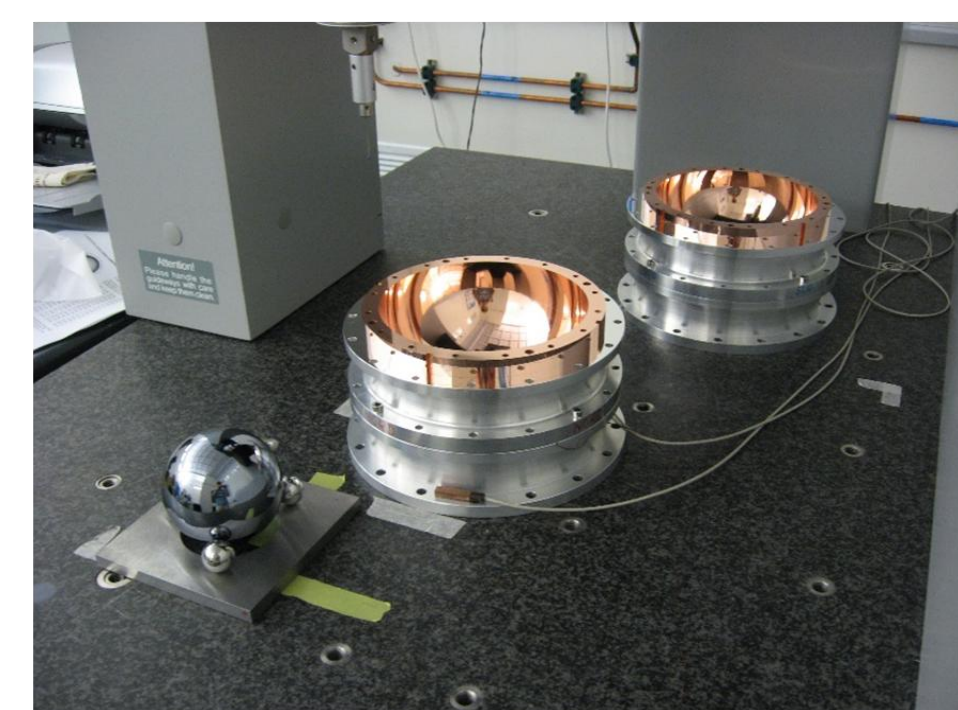
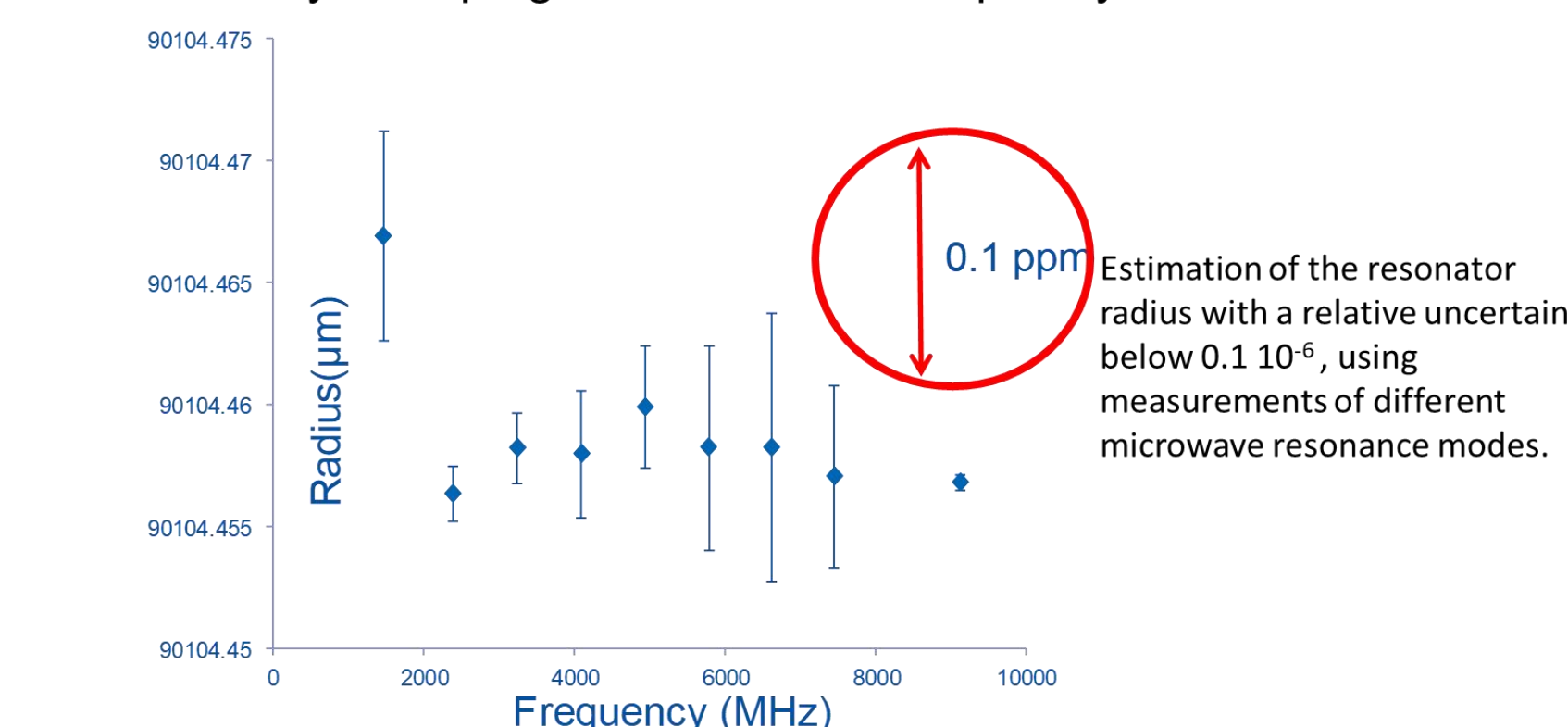
The 3 litre quasi sphere

- Diamond turned copper surface
- Triaxial ellipsoide
- Spherical outer surface
- 2 alternative ways to suspend the sphere (rigid or flexible)
- Conical plugs for reproducible insertion and removal of tubing, microphones and microwave antennæ.

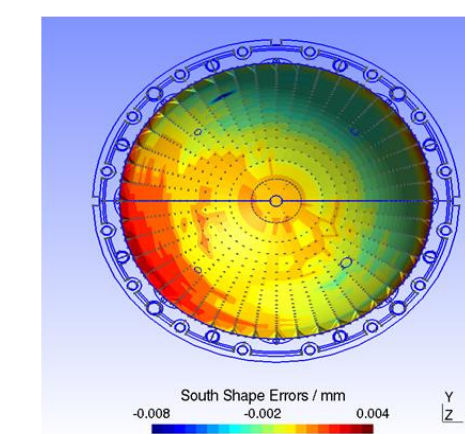
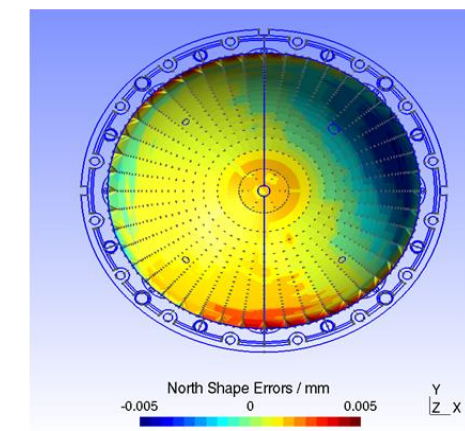


2: Determination of the radii

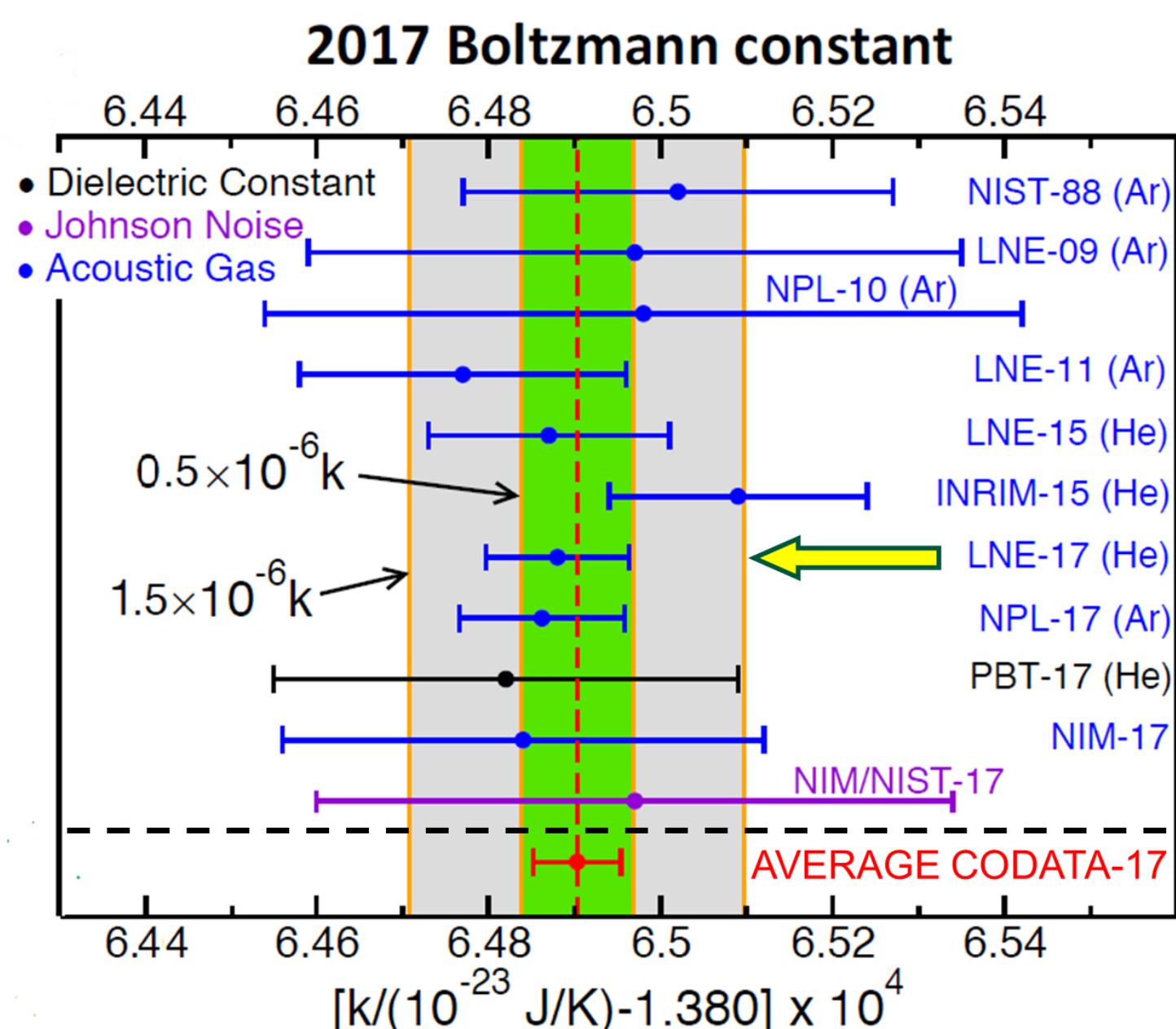
Measurement of microwave resonance frequencies used to determine the three different diameters of the quasi-spherical resonator. The geometry was deliberately non-spherical so as to lift the degeneracy of modes that would result in any case from imperfections were a perfect sphere chosen instead. The diameters are 90.074 mm, 90.134 mm and 90.194 mm. Loop antennæ are used to excite both TE and TM modes, which is important since these have different boundary conditions on the surface of the resonator. We recorded frequency responses of the resonator by sweeping the microwave frequency



A coordinate measuring machine gave an independent value of the volume and the shape in very good agreement with the microwave result.



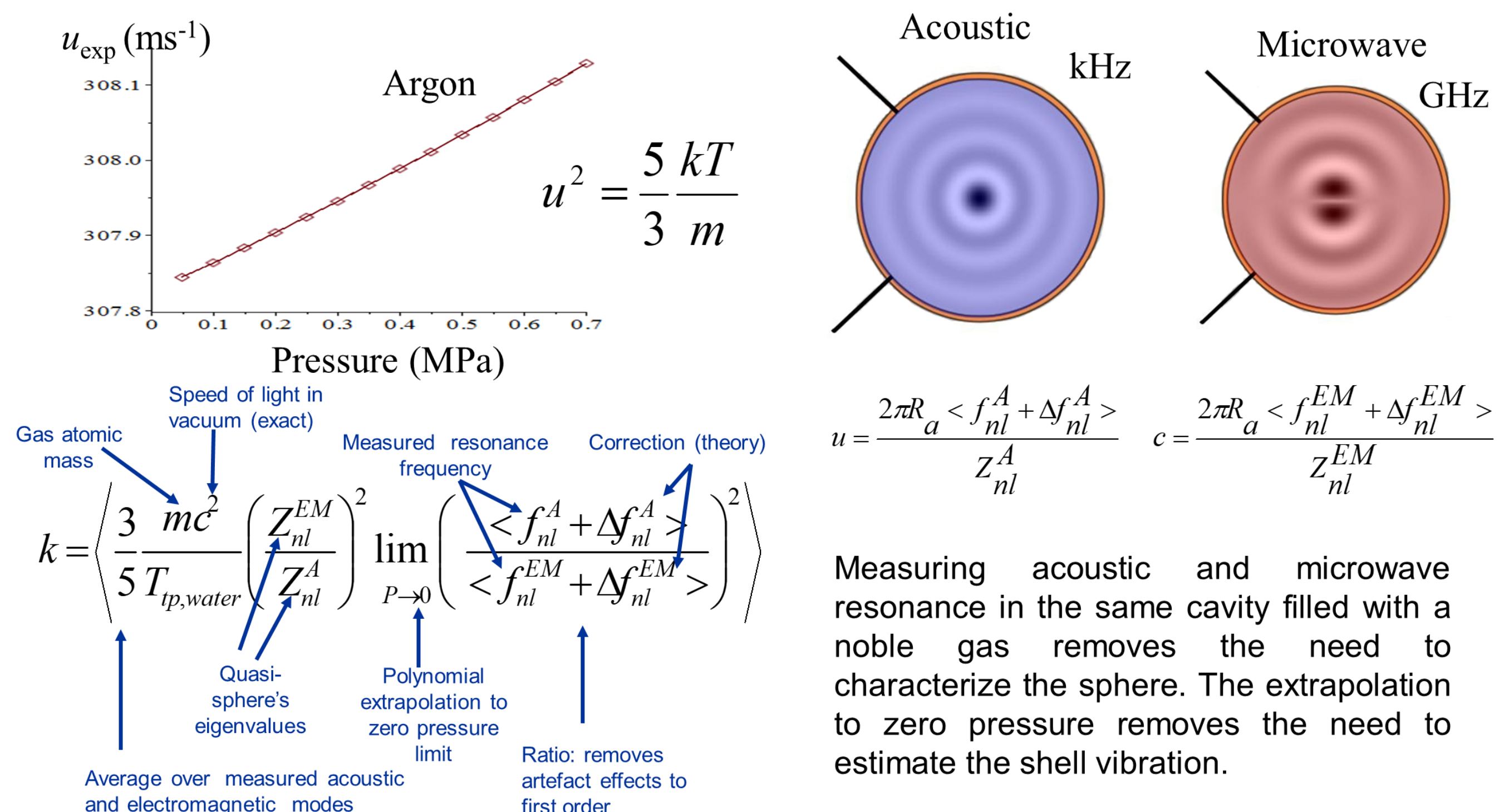
Final Measurements of the Boltzmann Constant Worldwide



LNE final value
 $k=1.380\,648\,78(83)\times 10^{-23}\text{J/K}$
2019 fixed value
 $k=1.380\,649\times 10^{-23}\text{J/K}$

Acoustic Gas thermometry (AGT) is the method which gives the smallest uncertainty in the determination of the Boltzmann constant k . Key: DC/RIGT dielectric constant/refractive index gas thermometry, JN Johnson noise thermometry.

The LNE-Cnam method



Confidence in the LNE Measurement of the Boltzmann Constant

At LNE-Cnam, a two-year experiment was necessary to establish a complete uncertainty budget consisting of more than 22 different elements.

Uncertainty budget for the thermometry measurements for the determination of the Boltzmann constant at the triple point of water T_{TPW}. The dominant contribution arises from the dispersion of thermometer readings at different locations.

Term	Uncertainty in k (parts in 10 ⁹)	Note
Repeatability and self-heating	0.08	Evaluated with measurements performed on the resonator
Standard resistor stability	0.05	Temperature stability and time-drift of the standard resistor
Resistance bridge	0.04	Bridge linearity and bandwidth
Dispersion of thermometer readings	0.31	Standard deviation of the temperatures measured on the resonator (scaled to T_{TPW}), includes the contribution of the correction ΔT_{TPW}
TPW calibration	0.21	Calibration at the TPW, including TPW repeatability, isotopic effect, purity, hydrostatic head effect and heat fluxes
TOTAL	0.39	Square root of the sum of squares

Uncertainty budget for molar mass determination. The relative change in the molar mass gives an identical relative change of the Boltzmann constant k .

Term	Effect on k (parts in 10 ⁹)	Note
³ He/ ⁴ He ratio	0.01	Uncertainty estimate from CRPG-CNRS
Impurity	0.02	Change of two bottles
Estimated outgassing	0.08	Effect of gas flow change
Single cold trap effectiveness	0.03	Supplementary study using Ar seeded He gas.
Difference of two cold traps	0.03	With or without charcoal
Total	0.09	Square root of the sum of squares

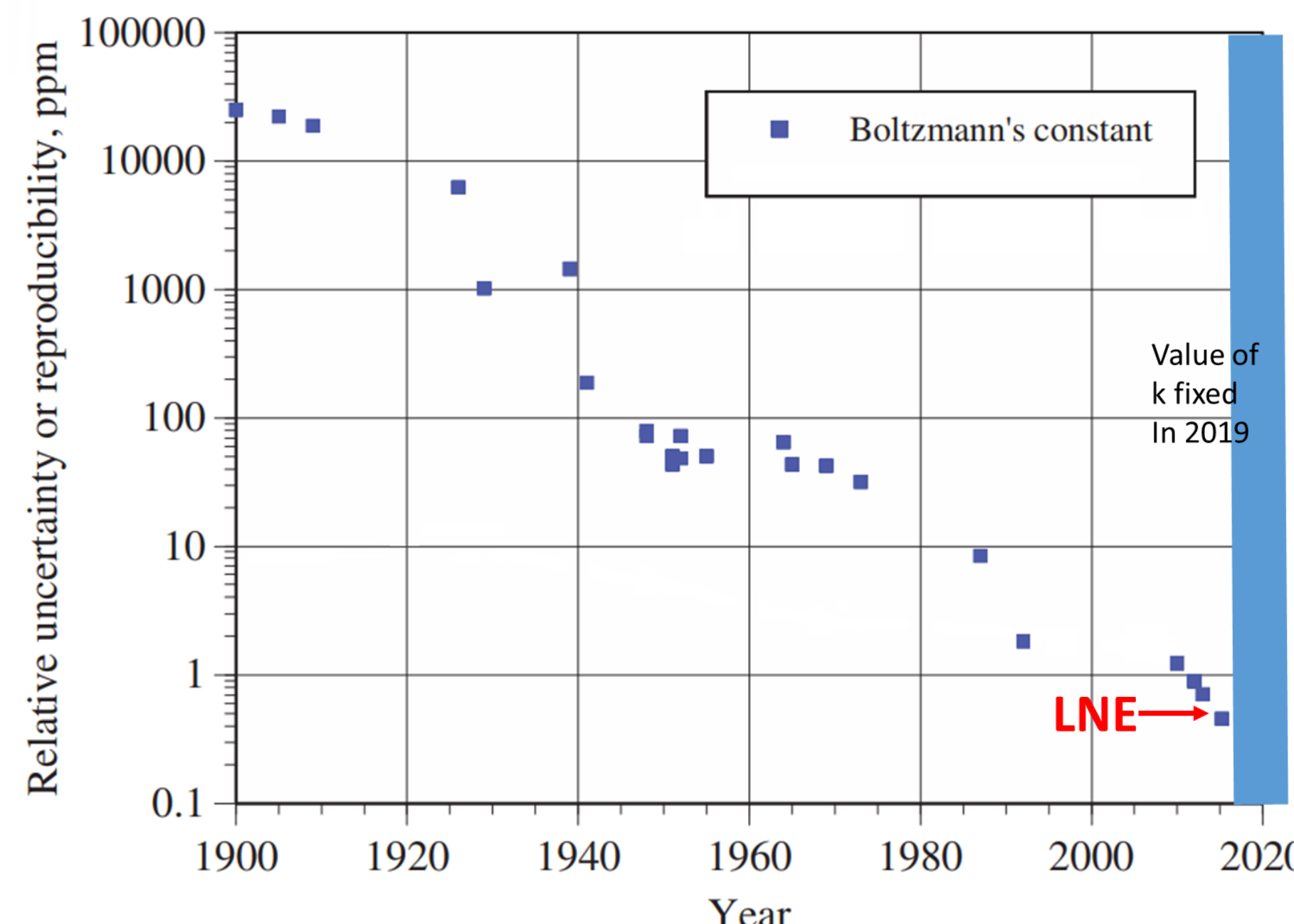
Uncertainty budget for the microwave determination of resonator dimensions. For detailed explanations of the components see Pitre *et al* which uses the same technique applied to a resonator of volume 0.5 L.

Uncertainty component	Effect on k in parts in 10 ⁹	Note
Resonance fit	0.008	Fitting uncertainty
Extrapolation to zero pressure	0.008	Uncertainty intercept of extrapolation
Scatter among microwave radii (includes uncertainty of shape perturbation)	0.08	Agreement between the TE and TM modes
Frequency reference	0.0	<1 part in 10 ¹⁰
Temperature calibration	0.009	
Temperature gradient	0.009	
Surface conductivity	0.16	Scatter of the average conductivity relative to the conductivity obtained using the TE ₁₁₂ mode
Microphone perturbation	0.023	Shift between the radii obtained with the TM ₁₁ mode and that of the other modes at the time of the first measurement and currently.
Waveguide corrections	0.05	Difference between the radii observed with straight and loop antennæ
Dielectric layer	0.067	Difference in the radii obtained with only TM modes and only TE modes
Total	0.20	Square root of the sum of squares

Term	Effect on k (Parts in 10 ⁹)	Note
Scatter among modes	0.28	Disagreement among modes (0.3) to (0.6) and (0.8)
Thermal conductivity of helium	0.02	Uncertainty of <i>ab initio</i> speed of sound
Uncertainty of <i>ab initio</i> speed of sound	0.06	Due to uncertainty in <i>ab initio</i> virial coefficients
Microphone perturbation	0.24	Worst possible case
Effect of gas flow change	0.02	Microphone or CMM eigenvalue
Relative shape perturbation corrections	0.09	Difference in result when the impedance is increased by 10%
Tubing acoustic impedance	0.09	Offset of 11.2 Pa
Pressure uncertainty	0.10	
Total	0.40	Square root of the sum of squares

k uncertainty over a century

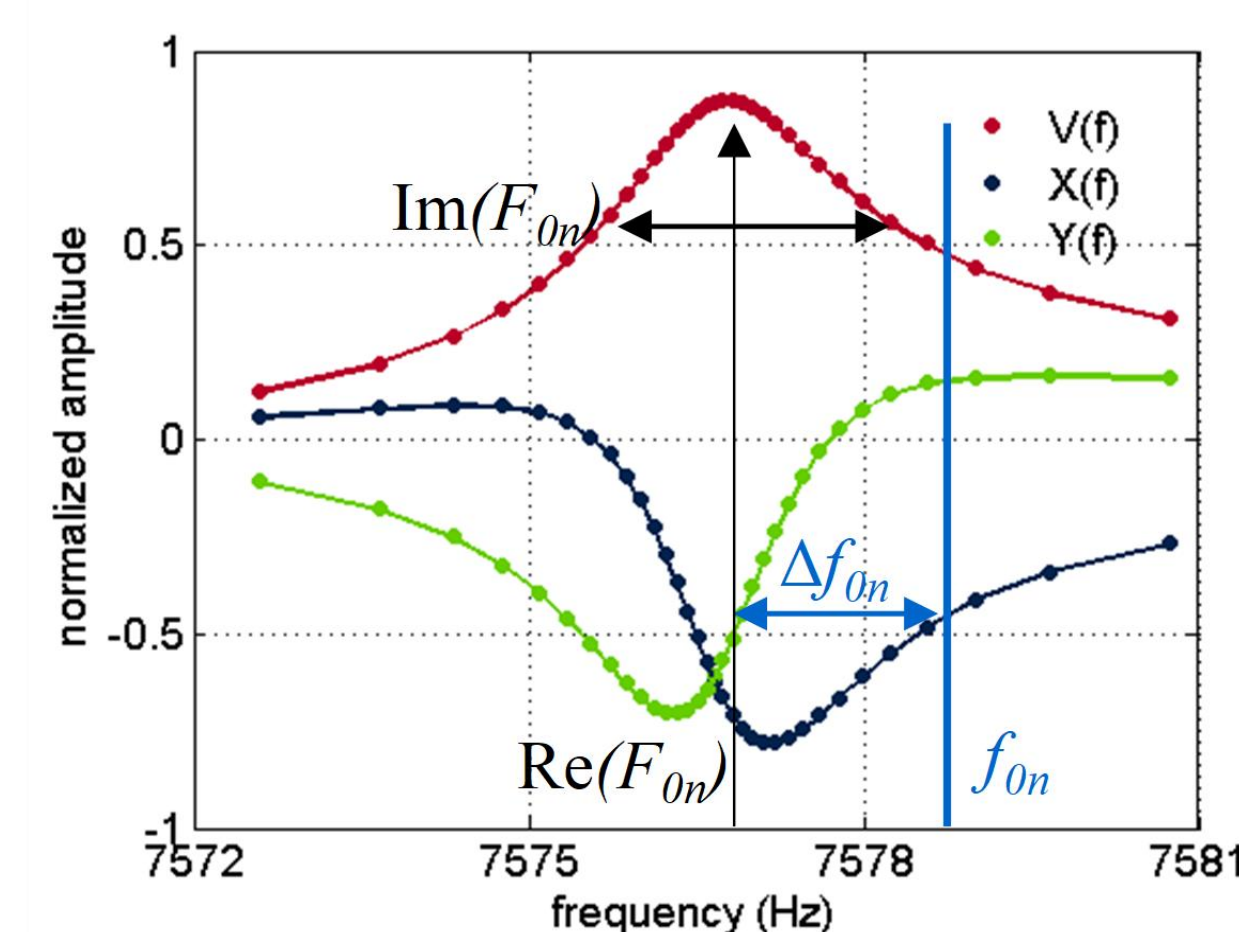
The LNE-Cnam's value for k was adopted by the CODATA Task Group on Fundamental Physical Constants in the set of values to be averaged, representing some 55% of the total weight in the adjustment.



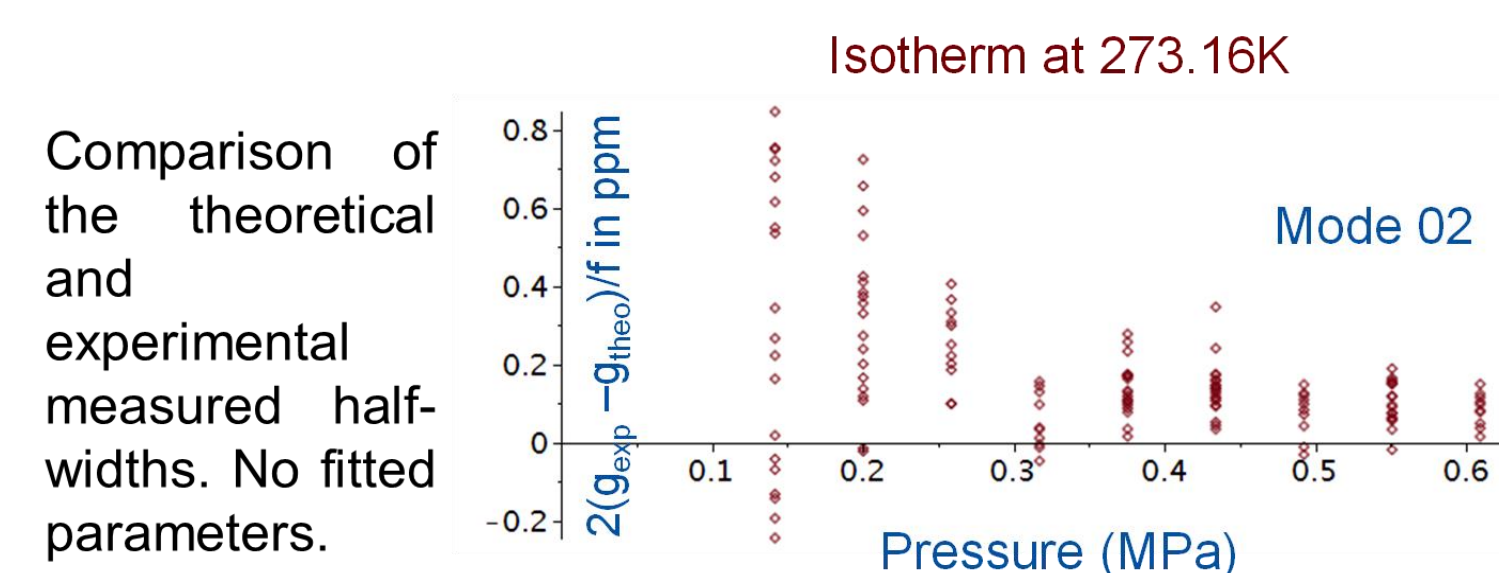
Evolution of the uncertainty of the Boltzmann Constant estimation and measurement over a century

3: Determination of the speed of sound

The oscillator frequency was scanned over the resonance frequency in both directions, with a non-uniform step spacing, to optimize acquisition time and line fitting accuracy. An 8-parameter model was fitted to the real and imaginary parts of the spectrum, which includes some cross talk and the effect of neighbouring resonances. Eight radial modes were measured from 2.44 kHz to 14.5 kHz. The quality factor of resonances range from 13 000 to 58 000, twice as high as in the work on the 0.5 L resonator. In that work, centre frequencies were measured to 10⁻⁷. The boundary layer correction represents 75 to 150×10⁻⁸ for the first radial mode and 34 to 72×10⁻⁸ for the last one. To extract the speed of sound from the value of the acoustic resonance frequencies, we derived the dimensions of the resonator from the measurement of its microwave resonance frequencies.

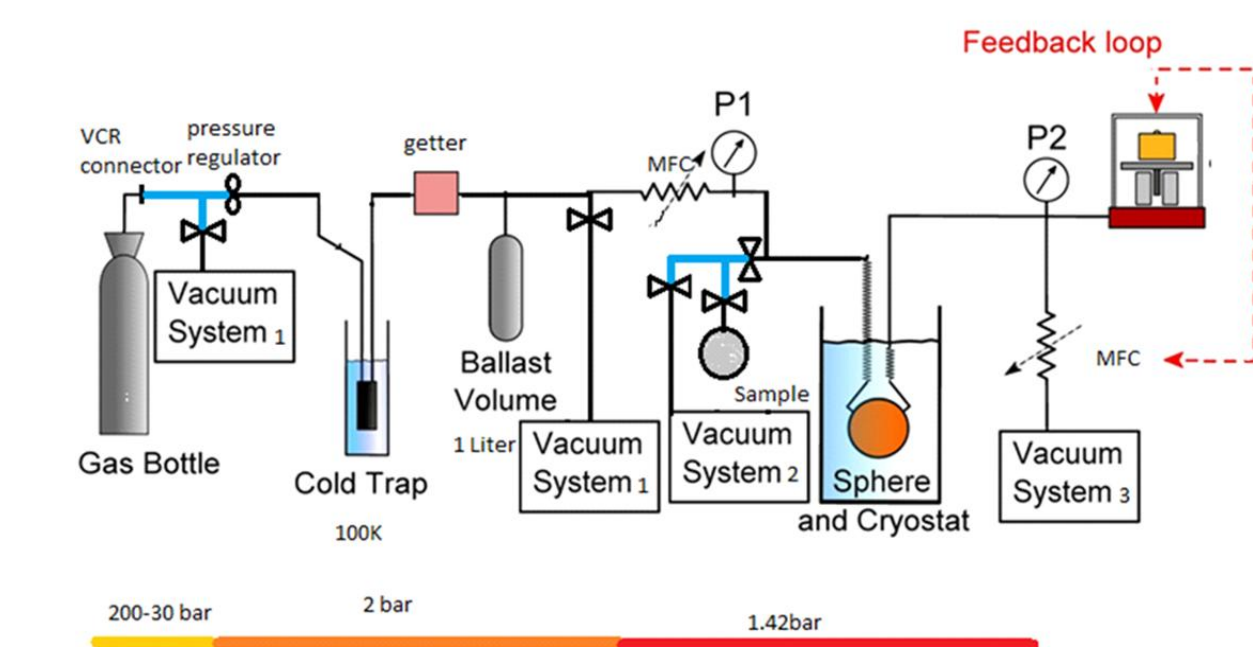


Determination of the frequency resonance and its half width

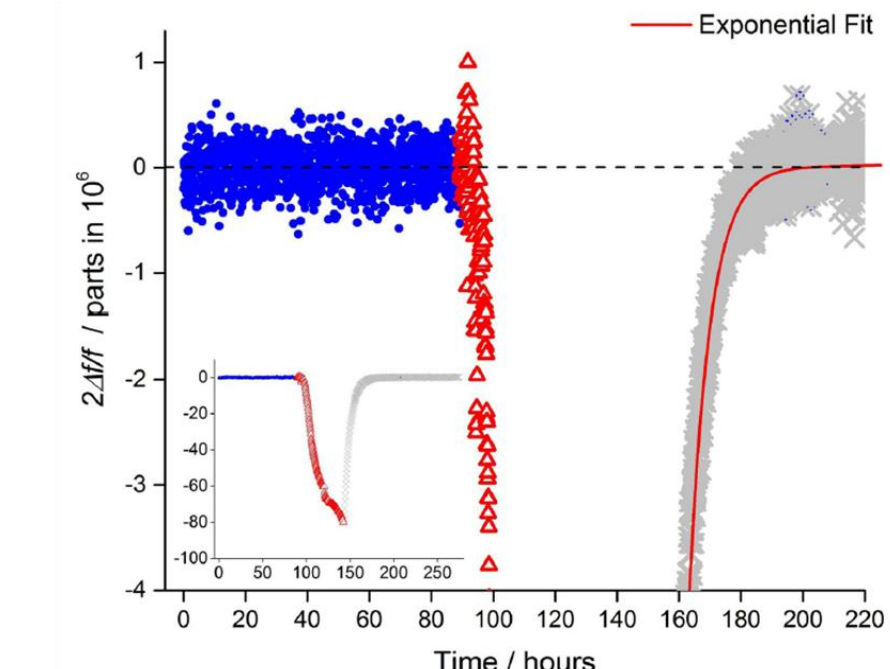


Comparison of the theoretical and experimental measured half-widths. No fitted parameters.

4: Gas handling



Gas handling system to control the pressure and the purity of the gas



Measurement of the effectiveness of a liquid-helium-cooled trap for removing argon gas. The square of the speed of sound is directly related to the square of the frequency of a given acoustic mode. On the left (blue), helium-4 gas is purified normally using the cold trap. In the middle (red), the cold trap is bypassed, allowing argon gas to diffuse into the system, causing a clear frequency shift of approximately 85 parts per million. On the right (grey), the cold trap is reintroduced, demonstrating recovery of the original conditions.

Future

- Dissemination of thermodynamic temperature in science and industry
- New pressure standards (Quantum Pascal)

See

Pitre L, Sparasci F, Risegari L, Guianvarc'h C, Martin C, Himbert M E, Plimmer M D, Allard A, Marty B, Giuliano Albo P A, Gao B, Moldover M R and Mehl J B *Metrologia* **54** 856-873 2017

Pitre, L., Plimmer, M. D., Sparasci, F., & Himbert, M. E. (2019).. *Comptes Rendus. Physique*, 20(1-2), 129-139.